DISCUSSION PAPER SERIES

No. 10969

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INDUSTRIAL ORGANIZATION

ISSN 0265-8003

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Discussion Paper No. 10969
December 2015
Submitted 23 November 2015

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INFORMATION, AUTHORITY, AND SMOOTH COMMUNICATION IN ORGANIZATIONS†

Abstract

Two divisions of a firm, overarched by a headquarters, are engaged in a decision problem. Division one obtains information and informs division two who has the formal authority to make the decision. Headquarters guides the decision process by affecting the quality of information that division one obtains. In equilibrium, division one honestly communicates the inferences drawn from its observations, but not the underlying observations themselves and division two takes the advice at face value. The communication equilibrium involves smooth strategies and is outcome equivalent to delegation: the informed party gets its way, regardless of the allocation of formal authority.

JEL Classification: D82
Keywords: authority, delegation, endogenous information and strategic communication

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† This paper is a spin-off of an earlier version, entitled “Smooth, strategic communication". We thank Ricardo Alonso, Patrick Bolton, Wouter Dessein, Peter Eső, Emir Kamenica, Botond Kőszegi, Matthias Lang, David Martimort, Meg Meyer, Benny Moldovanu, Inés Moreno de Barreda, Marco Ottaviani, Sven Rady, Joel Sobel, Peter Sørensen, Peter Vida, and seminar participants at the Universities of Aarhus, Bonn, CEU Budapest, Copenhagen, Oxford, PSE; SYME 2013 (Aarhus), ESEM 2014 (Toulouse), ESSET 2015 (Gerzensee), VfS 2014 (Hamburg), and the SFB-TR15 conference 2014 (Mannheim). Financial support by the DFG in form of a SFB-TR15 grant and the BGSE is gratefully acknowledged. All remaining errors are our own.

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Keywords: authority, endogenous information, strategic communication, delegation

1 Introduction

Sound decision making requires good information. The success of organizations depends crucially on the quality of information their decision-makers have and on the alignment of interests within the organization. Most of the time, organizations do not have automatic access to information but must actively acquire it prior to decision-making. The search for information is subject to choices and must be considered as part of the decision-making process. How and where the information enters the organization is by and large determined by the organization's existing structure. Inside the organization the information needs to be communicated to the decision-maker. Such communication is prone to strategic manipulation; on the way towards the decision-maker, inferences are drawn, details can be dropped, things can be swept under the rug. The present paper tries to shed light on how organizations with given communication channels can cope with such problems. We show that an appropriate acquisition of information can ensure sound decision-making despite strategic communication, provided that a priori known conflicts are eliminated.

Our way to demonstrate this result builds on the following insights. Conflicts within the organization depend critically on the information available. As a result of this, feeding better information into the organization does not necessarily imply better decision-making; it may instead result in more relevant things being swept under the rug. Due to their ability to withhold information, those who can filter information along its way have a significant influence on the decisions that are made. Rational information acquisition by the organization takes all these factors into account and eliminates conflicts to the point where this is possible. As a consequence, based on the information that reaches the decision-maker, all parties would make the same decision; communication and delegation are outcome equivalent. In other words, everything is as if the informed party were formally legitimized to make choices.

Our conclusions stem from an analysis of a stylized model of a multidivisional organization with a common headquarters. For concreteness, we have in mind a car manufacturer with a European and a US-American division. The firm wishes to build a new model that will be sold on both markets. To learn about customers' tastes, the firm launches a market study. Due to economies of scale, market research is directed by the US division, production is directed by the European division. The tastes of Europeans and US-Americans are positively but not perfectly correlated. No systematic taste differences between the continents are expected; prior conflicts are absent. But, depending on the results of the study, different designs could turn out to be optimal for each market and conflicts could arise between the divisions ex post. Hence, communication is strategic and prone to manipulation.

We show that at most the inferences from the study, the optimal design from the US-division's perspective, but not the observed results themselves can be communicated in equilibrium. What inferences the European receiver infers from the US sender's inference depends critically on the nature of information that enters the organization. Headquarters shapes the communication process with a view to reaching the highest feasible joint surplus for the two divisions. Its only influence is through the attention devoted to the two markets in the study. The optimal way to do this is to equalize the residual uncertainty that remains for each division when information is used optimally from the receiver's point of view. Communication is not completely honest, as details are dropped, but unbiased: based on the optimal information, the sender's recommendation is an unbiased estimate of the receiver's preferred course of action. To achieve this unbiasedness, the market study devotes relatively more attention to the receiver's market. The sender needs to be forced to base his inference more heavily on receiver relevant facts and so sender relevant facts need to be observed with noise. Since optimal information eliminates biases, it does not matter where the decision is taken; it is always the same. Moreover, as the sender transmits the maximal amount of information he is willing to provide, the mechanism reaches the highest payoff for the organization among all direct communication protocols where the sender gets to see the information first. Hence, decision-making is arguably sound, as claimed.

Several lines of thought in our theory appear already, without a formal model, in March and Simon (1958). In their description of problem-solving, the authors note that: "The design of the search process is itself often an object of rational decision." (p.140). In their discussion of communication processes inside an organization, the authors coin the term *uncertainty absorption* and describe its consequences as follows:

> "Uncertainty absorption takes place when inferences are drawn from a body

> of evidence and the inferences, instead of the evidence itself, are then communicated. [...] Both the amount and the *locus of uncertainty absorption affect the influence structure of the organization.* Because of this, uncertainty absorption is frequently used, consciously or unconsciously, as a technique for acquiring and exercising power. [...] Whatever may be the position in the organization holding the formal authority to legitimize the decision, to a considerable extent the effective discretion is exercised at the points of uncertainty absorption."
> (March and Simon (1958), pp 165–167, emphasis in original)

In our model, uncertainty absorption corresponds to the sender drawing a unidimensional inference - a conditional expectation - from multiple signals. And indeed, although the receiver is formally legitimized to make the decision, the effective discretion is in fact exercised by the sender. This goes so far that communication and delegation become outcome equivalent. Given optimal information, allocating formal authority to the informed sender or bringing the information to the receiver are two ways to reach exactly the same outcome.

Our opening lines are inspired by the picture of organizations drawn by Cyert and March (1963), in particular their insightful discussion of communication and information acquisition (chapter 4). The ideas that information needs to be acquired, that the search for information is endogenous, and that the communication system influences the information that is acquired, all appear in their work. Our contribution is to offer a formal model that puts these elements together and hopefully advances our understanding of them. Our main result is that decisions can be steered indirectly by choosing what issues to look into and how deeply to probe into them. While it may be surprising how well this works in principle, it seems obvious that it does work in practice. Indeed, Cyert et al. (1958) offer case study evidence consistent with our theory. The authors followed a medium-large manufacturing concern in the 1950s in the process of installing an electronic data-processing system. It was quickly decided that an outside consulting firm was needed. An offer was obtained from a consulting firm named Alpha in the study. There was an important person in the manufacturing concern, named the controller. After Alpha had made its offer, the controller decided that a competing offer should be requested from another firm; he selected a firm Beta out of a list of candidates that had been prepared beforehand. Beta delivered its offer. A memorandum was prepared at the request of the controller that listed the criteria that should be looked at to compare the offers and reach a decision. The final staff memorandum on the decision clearly recommended to hire Beta, a recommendation that the controller accepted. The controller is cited with the words: "I asked the boys to set down the pros and cons. The decision was Beta. It was entirely their decision."(Cyert et al. (1958), p.332)

Of course, we will never know why the boys favored Beta over Alpha; it could be that they wanted to please the controller or that Beta made the better offer. However, there is no account of explicit manipulation in the study. The point is that the controller can steer the decision indirectly to the point that it doesn't really matter who takes the decision. Our paper shows that this is precisely how a benevolent controller should act.

We are not the first to take up Simon's concept of authority. Aghion and Tirole (1997) distinguish formal from real authority. The allocation of formal authority has important effects on initiative and participation when there are private costs of information acquisition. In contrast, we abstract from such costs and information is acquired by the organization itself. On top of this, our concept of real authority is different, allowing the receiver to amend proposals as in Crawford and Sobel (1982), the seminal paper on strategic information transmission between a sender and a receiver. Dessein (2002) studies the allocation of formal authority in the Crawford-Sobel model and shows that delegating decision rights to the informed sender is always better than communicating whenever meaningful communication is possible at all. The essential differences to the present paper are the nature of biases and information. In the Crawford-Sobel model, the sender wishes to induce an action that exceeds the ideal action of the receiver in each state of the world by some constant. Moreover, the sender's information is exogenously given. In our model, the information is endogenously determined by the organization and influences the magnitude and direction of biases, which both depend on the realized state of the world. If the organization can adapt to the situation along the informational margin, then delegation and communication become perfect substitutes.

Alonso et al. (2008) study the allocation of formal authority in an organization where two divisions interact with a headquarters. Both divisions have some information and need to make choices, preferably in a coordinated way. The organization can choose between vertical communication where all information flows upwards to a headquarters or horizontal communication where one division communicates with the other and the latter is in charge of decision making for both divisions. Depending on the relative importance of coordinating actions and of adapting choices to local conditions either one or the other form of communication is optimal.[1] We study the same organization, however, in a quite different situation where the form of the organization is exogenously given and information instead is endogenous. Allowing the firm to choose the information that enters the organization makes different allocations of formal authority perfect substitutes in our model.

Communication works so well in our model, because the organization acquires information that eliminates conflicts to the point where this is possible. Although communication is not completely honest about the observed evidence, it is honest about the inference drawn from the evidence. Following the sender's advice one-for-one is optimal for the receiver, because remaining conflicts are orthogonal to the sender's recommendation. Battaglini (2002) studies a multi-sender multidimensional cheap talk problem and uses an orthogonal construction to elicit perfect information from the senders. Although we rely on orthogonality as Battaglini does, we cannot apply his construction because there is only one sender in our model and a unidimensional choice needs to be made. Instead, we need to adjust the information that the sender obtains to ensure orthogonality.[2] Preferences over information are not studied in Battaglini (2002).[3]

Controlling the access to information in a communication game is first studied in Ivanov (2010), showing that communicating with an expert who has partial information is better for the receiver than talking to a an expert who is perfectly informed whenever meaningful communication is possible. Moreover, communication with controlled information can even

[1] An important difference between Dessein (2002) and Alonso et al. (2008) and the current paper are that biases are state dependent in the latter. Such biases have also been analyzed, e.g., by Stein (1989), Ottaviani and Sørensen (2006a), Ottaviani and Sørensen (2006b), Kawamura (2015), and in the most general model by Gordon (2010).

[2] In a model with a privately known bias, Li and Madarász (2008) remark that communication works well if the bias is independent of the state and symmetrically distributed around zero. However, these authors study mandatory disclosure of given biases, whereas biases arise from information in our model. For further analyses of privately known unidirectional biases, see Morgan and Stocken (2003) and Dimitrakas and Sarafidis (2005).

[3] For other approaches to multidimensional cheap talk, see Meyer et al. (2013), Chakraborty and Harbaugh (2007), Chakraborty and Harbaugh (2010), and Levy and Razin (2007). These papers are not concerned with the impact of the quality of multidimensional information on communication.

outperform optimal delegation to a perfectly informed expert from the receiver's point of view. The common ground with the present paper is the comparison of institutions, one of which involves controlling the quality of information, broadly speaking.[4] However, there are substantial differences, the most important one is that Ivanov (2010) analyzes senders who are systematically biased in one direction. Moreover, we study noisy information structures within a class that induces smooth posteriors whereas Ivanov (2010) investigates partitional information structures. Our main result is the outcome equivalence of optimal delegation and communication, which does not arise in Ivanov's model.[5]

The optimal information structure is noisy in our model, in order to make the sender willing to share his information and the receiver willing to use it; so, noise helps to facilitate communication as in Blume et al. (2007) or Goltsman et al. (2009). However, in Blume et al. (2007), the sender has perfect information and noise is added to the sender's message, while our sender is endowed with noisy information but communicates without further noise. Goltsman et al. (2009) compare the outcomes of different decision protocols and show, among other results, that the noise-mechanism of Blume et al. (2007) is an optimal mediation mechanism. Moscarini (2007) assumes Gaussian noisy information and noiseless communication to study central bank competence. Communication equilibria are partitional in his analysis and information is exogenously given in his approach; our comparative statics predictions are similar. Gordon and Nöldeke (2013) combine Gaussian noise in information and communication. Similar to our paper, the communication equilibria are in linear strategies. However, Gordon and Nöldeke (2013) restrict attention to the class of equilibria in linear strategies a priori and use the resulting equilibrium strategies to explain figures of speech, such as exaggeration, understatement, and irony. The existence of these equilibria depends on the noise that is added exogenously to the sender's message. In contrast, communication is noiseless in our model and we are interested in an unrestricted optimum of our game.

[4] Argenziano et al. (2013) compare delegation and communication when the sender has a one-sided bias and acquires costly information.

[5] A further difference is that Ivanov (2010) studies information structures that are optimal for the receiver whereas we study optimality from the perspective of joint surplus. For an analysis of sender optimal information structures, see, e.g., Szalay (2005) and Eső and Szalay (2015). Kamenica and Gentzkow (2011) analyze sender optimal persuasion rules; the difference to the present problem is the commitment to information that reaches the decision-maker.

Preferences over information in markets have been studied extensively. Vives (1999) surveys the literature on product market competition with information frictions, Vives (2008) the literature on financial markets. A more recent overview is given in Pavan and Vives (2015). Angeletos and Pavan (2007) investigate the social value of information in large markets with strategic complementarity or substitutability, externalities, and heterogeneous information. This literature relies on Gaussian noise and, depending on the context, either CARA preferences or quadratic payoffs, to find equilibria in linear strategies. Nöldeke and Tröger (2006) prove the existence of linear strategy equilibria in a market microstructure model for the wider class of elliptical distributions, which contains the Normal distribution as a special case. We allow at the same time for general payoff functions and elliptical distributions. We are not aware of any other contribution that does so too. Note also that in the literature on strategic market interactions, agents observe information and choose actions directly. Cheap talk communication of unverifiable information followed by a common action that affects the payoffs of a sender and a receiver is not analyzed in this literature.

The remainder of the paper is organized as follows. In section two, we present the model. In section three, we analyze communication and derive an upper bound on the amount of information that can be transmitted in any equilibrium. In section four, we analyze optimal information acquisition from the organization's perspective. A final section concludes and discusses extensions. Lengthy proofs are gathered in the appendix.

2 Model

We consider a firm, comprised of two divisions with a common headquarters. A decision $x \in \mathbb{R}$ needs to be taken that affects the payoffs of all three parties. Division one has preferences described by

$$u^S(x, \eta) = -\ell(x - \eta);$$

division two has preferences

$$u^R(x, \omega) = -\ell(x - \omega).$$

The loss function $\ell(q)$ is symmetric around its minimizer, $q = 0$, twice differentiable, and at least as convex as the quadratic function. More precisely, we assume that the Arrow-

Pratt measure of relative curvature of the loss function satisfies $\frac{q\ell''(q)}{\ell'(q)} \geq 1$ for all $q \neq 0$.[6] In addition, ℓ rises sufficiently slowly to make expected utility well-defined. η and ω are random variables - the tastes of consumers that are served by the two divisions - whose realizations describe the ideal policies from each division's point of view. These ideal policies are given by $x^R(\omega) = \omega$ and $x^S(\eta) = \eta$, respectively. The realizations of ω and η are unknown at the outset. Headquarters is interested in joint surplus[7]

$$u^H(x, \eta, \omega) = -\ell(x - \eta) - \ell(x - \omega).$$

The decision process in the firm is organized as follows. Division one, henceforth the sender, gets to observe noisy signals

$$s_\omega = \omega + \varepsilon_\omega \quad \text{and} \quad s_\eta = \eta + \varepsilon_\eta,$$

where ε_ω and ε_η are uncorrelated noise terms. Division two, henceforth the receiver, is in charge of making the decision. Headquarters shapes the communication between the divisions by controlling the research that division one conducts. Formally, headquarters chooses the amount of noise in the sender's signals, that is the variances $\sigma^2_{\varepsilon_\omega}$ and $\sigma^2_{\varepsilon_\eta}$ of the noise terms ε_ω and ε_η. This choice is publicly observable. However, the realizations of signals s_ω and s_η are privately observed by the sender. The sender communicates with the receiver, who finally chooses x. There is no cost of sending messages and the receiver is unable to commit to the action x as a function of the information he receives, so communication is modeled as cheap talk in the sense of Crawford and Sobel (1982).

To make the updating about the underlying states tractable we place restrictions on the joint distribution of $\omega, \eta, \varepsilon_\omega$ and ε_η. We focus on an environment where conditional means are linear functions of the observed information. Moreover, linear transformations of the underlying random variables follow the same class of distribution as the underlying random variables do. As is well known, these assumptions are satisfied, e.g., if $\omega, \eta, \varepsilon_\omega$ and ε_η are jointly normally distributed. However, these assumptions are generally fulfilled by all members of the class of elliptical distributions, which includes the Normal distribution as a

[6] Examples include $\ell(q) = q^{2n}$ for $n \in \mathbb{N}$.

[7] As shown by Alonso et al. (2008), profit sharing between headquarters and the divisions gives rise to such headquarters preferences.

special case. In what follows, we term the joint distribution of ω, η, s_ω and s_η the *information structure*. An information structure is feasible if it belongs to the elliptical class, has a density function, finite first and second moments, and if the marginal joint distribution of ω and η equals the prior distribution. Given these assumptions, the joint density of a random vector $\mathbf{Y}$ of dimension n can be written as $f_{\mathbf{Y}}(\mathbf{y}) = c_n |\mathbf{\Sigma}|^{-\frac{1}{2}} \phi\left((\mathbf{y}-\mu)' \mathbf{\Sigma}^{-1}(\mathbf{y}-\mu)\right)$, where μ is the mean vector, $\mathbf{\Sigma}$ is up to a constant factor equal to the covariance matrix, $\phi(\cdot)$ is a given function, and c_n a scale factor, which we simply denote $c = c_1$ in the one-dimensional case.[8]

We assume that all the differences in preferences are unsystematic and random. Formally, we assume that $\mathbb{E}[\omega] = \mathbb{E}[\eta]$. This amounts to saying that systematic differences in preferences - where one division wishes to push the decision in a particular direction relative to the other division's preferred choice - have been eliminated prior to the current interaction. This does not imply that preferences are aligned. It only implies that based on prior information no differences of opinions are expected. In addition, we impose the innocuous normalization that $\mathbb{E}[\omega] = \mathbb{E}[\eta] = \mathbb{E}[\varepsilon_\omega] = \mathbb{E}[\varepsilon_\eta] = 0$. The covariance matrix is described by $\sigma_\omega^2 \equiv Var(\omega)$, $\sigma_\eta^2 \equiv Var(\eta)$, $\sigma_{\varepsilon_i}^2 \equiv Var(\varepsilon_i)$ for $i = \omega, \eta$, and $\sigma_{\omega\eta} \equiv Cov(\omega, \eta)$. The covariances involving the noise terms are zero by assumption. The coefficient of correlation between ω and η is defined as

$$\rho \equiv \frac{\sigma_{\omega\eta}}{\sigma_\omega \sigma_\eta}.$$

To complete the description of the model, consider the ideal policies from each division's perspective if each of them had access to the information s_ω and s_η.

Lemma 1 *As functions of the underlying signal realizations, s_ω, s_η, the ideal choice functions of the receiver and the sender are*

$$x^R(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^R(x,\omega) \middle| s_\omega, s_\eta\right] = \mathbb{E}[\omega | s_\omega, s_\eta] = \alpha^R s_\omega + \beta^R s_\eta$$

and

$$x^S(s_\omega, s_\eta) \equiv \arg\max_x \mathbb{E}\left[u^S(x,\eta) \middle| s_\omega, s_\eta\right] = \mathbb{E}[\eta | s_\omega, s_\eta] = \alpha^S s_\omega + \beta^S s_\eta,$$

[8]The Normal distribution corresponds to the case $\phi(u) = e^{-\frac{u}{2}}$ and $\mathbf{\Sigma}$ identically equal to the covariance matrix. The factor c_n depends on n to make f a density. Other members of the elliptical class include, e.g., the exponential power distribution (and as a special case the Laplace) or the logistic distribution. For more details on elliptical distributions see, e.g., Fang et al. (1990).

where α^i, β^i *for* $i = R, S$ *are weights, independent of* s_ω, s_η.
Unless $\sigma_\omega^2 = \sigma_\eta^2 = \sigma_{\omega\eta}$, $x^R(s_\omega, s_\eta) \neq x^S(s_\omega, s_\eta)$ *for all* $s_\omega, s_\eta \neq 0$.

The optimal choice functions correspond to the conditional expectations and conditional expectations are linear in our statistical framework. The intuition is familiar from the Normal distribution-quadratic loss case; we state the result as a lemma, because we prove the generalization both with respect to a wider class of distributions and loss functions.

The divisions disagree on the optimal course of action for almost all signal realizations unless the tastes of their customers are perfectly correlated with identical marginal distributions, in which case their customers are essentially identical. The coefficient of correlation captures the alignment of interests in an intuitive way. It is easy to show that no meaningful communication is possible if $\rho \leq 0$.[9] To focus on the interesting case, we assume that $0 < \rho < 1$.

It is worth pausing for a minute to discuss the crucial assumptions and differences to other approaches in the literature. The main difference is the way we capture conflicts of interests. We assume identical loss functions for sender and receiver and capture all the differences between them by the random variables ω and η and their distributions. The first moments describe ideal policies, the second moments shape expected utilities. Assuming equal prior expectations amounts to saying that differences of opinion prior to the current interaction have been eliminated. The remaining conflicts are random and unsystematic, in the sense that their expected value is zero. We make these assumptions, because it is by now well known that communication does not work well with systematic differences of opinions. In contrast, it is not yet known how well communication can work with unsystematic differences of opinions.

We analyze the game proceeding backwards, starting with the inference that the sender draws from observing facts and the ensuing communication continuation games. We then reduce the model to one where communication is about inferences instead of facts and discuss the receiver's inferences drawn from the sender's inference. Building on this analysis, we discuss the optimal organizational response to filtering information this way, the optimal amount and kind of information that the organization acquires.

[9]A formal proof of this statement is available from the authors upon request.

3 The sender as a strategic information channel

Suppose that headquarters has chosen a research policy - formally, an information structure - and the sender gets to observe the results of the research. What part of the observed information is the sender willing to share with the receiver at all?

3.1 Limits to communication

We focus on Bayesian equilibria in the communication game. After observing signal realizations s_ω, s_η, the sender sends a message $m \in \mathbb{M}$ to the receiver. The message space is sufficiently rich; we do not impose any restrictions on $\mathbb{M}$. It is enough to consider pure message strategies for the sender.[10] A pure sender strategy maps the sender's information into messages $M : \mathbb{R}^2 \to \mathbb{M}$, $(s_\omega, s_\eta) \mapsto m$. A pure receiver strategy maps messages into actions, $X : \mathbb{M} \to \mathbb{R}$, $m \mapsto x$. The receiver updates his belief about the sender's type after observing the sender's message and acts optimally against this belief. The following lemma derives an upper bound on the information that can be communicated in any equilibrium of the communication game. In particular, the sender is willing to share his inference but not the underlying facts.

Lemma 2 *In any equilibrium, all sender types s_ω, s_η such that $\alpha^S s_\omega + \beta^S s_\eta = constant$ induce the same action.*

Define the statistic

$$\theta \equiv \alpha^S s_\omega + \beta^S s_\eta.$$

All sender types with signal realizations s_ω, s_η adding up to θ share the same ideal policy, θ. Moreover, with symmetric loss functions, the sender's preferences over distinct actions depend only on the distance of these actions to θ. Hence, the set of types who share the same θ induce at most two distinct actions, and these actions need to be equidistant from θ in any equilibrium. However, any attempt to separate sender types whose signals aggregate to θ into subsets that induce distinct actions gives some other types, whose signals aggregate

[10] More specifically, it is standard in the literature to look at the most informative equilibria and these equilibria involve pure strategies in our game. Therefore, we abstain from introducing the notational clutter to deal formally with mixed strategies.

to some value close to θ, a strict incentive to lie. Hence, no such equilibrium can exist. Obviously, the lemma also implies that it is impossible to elicit the information s_ω, s_η from the sender, unless the ideal policies of sender and receiver coincide altogether.

Corollary 1 *Truthful communication of the underlying information, s_ω, s_η, is an equilibrium if and only if $\sigma_\omega^2 = \sigma_\eta^2 = \sigma_{\omega\eta}$.*

Since induced actions depend only on the realization of θ, the sender is willing to reveal at most the inference he draws from the facts, that is θ, but never the underlying facts. Hence, we can characterize any equilibrium of the communication game in terms of communication about the sender's inference, θ, only.[11]

3.2 Inference from inference and conflicts

¿From the ex ante perspective, before the signals are realized, the sender's inference is random itself. Any given choice of information structure gives rise to a joint distribution of ω, η, and θ. Given that $\omega, \eta, \varepsilon_\omega$ and ε_η follow a joint elliptical (Normal) distribution, the random variables ω, η, and θ follow a joint elliptical (Normal) distribution as well.[12] One can show that the moments involving θ are given by $\mathbb{E}[\theta] = 0$ as well as

$$Var(\theta) = \sigma_\eta^2 \frac{\frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2} + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\rho^2 + 1 - \rho^2}{\left(1 + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}\right)\left(1 + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\right) - \rho^2}, \tag{1}$$

$$Cov(\omega, \theta) = \sigma_{\omega\eta} \frac{\frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2} + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2} + 1 - \rho^2}{\left(1 + \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}\right)\left(1 + \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}\right) - \rho^2}, \tag{2}$$

and

$$Cov(\eta, \theta) = Var(\theta). \tag{3}$$

Equations (1) and (2) depend crucially on the normalized noise variances, $\frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}$ and $\frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}$. By construction of θ, only covariance matrices with $Cov(\eta, \theta) = Var(\theta)$ are possible. For all

[11]Note the close connection between this result and the process of uncertainty absorption described in March and Simon (1958).

[12]The proof of this statement follows from Fang et al. (1990) Theorem 2.16.

that matters in terms of induced choices and payoffs, we can analyze our model in terms of this reduced form joint distribution of inference and underlying states.

What inference would the receiver draw if the sender communicated his inference? Since the joint distribution of ω, η, and θ is firmly within the class that has linear conditional means, the receiver's ideal policy conditional on observing θ is

$$\mathbb{E}\left[\omega|\,\theta\right] = \frac{Cov\left(\omega,\theta\right)}{Var\left(\theta\right)} \cdot \theta. \tag{4}$$

The conditional expectation corresponds to the linear regression of the unknown state on the observed information. To understand the slope of the regression, note that the regression of η on θ is simply

$$\mathbb{E}\left[\eta|\,\theta\right] = \frac{Cov\left(\eta,\theta\right)}{Var\left(\theta\right)} \cdot \theta = \theta. \tag{5}$$

Clearly, given that θ is the conditional expectation of η given the underlying facts, the sender does not revise his conditional expectation if shown θ again. In contrast, the receiver's inference corrects for the relative informational content of the sender's inference, θ, with respect to the underlying states ω and η: by equation (3), the slope $\frac{Cov(\omega,\theta)}{Var(\theta)}$ corresponds to $\frac{Cov(\omega,\theta)}{Cov(\eta,\theta)}$. If the sender gets to observe information that is relatively more informative about ω than about η, then $Cov\left(\omega,\theta\right) > Cov\left(\eta,\theta\right)$ and the receiver's ideal policy attaches a higher weight to the information θ than the sender's ideal policy. The situation is reversed if the sender gets to see information that is relatively more useful to the sender. The regressions have identical slopes if the sender's inference is equally informative about ω and η.

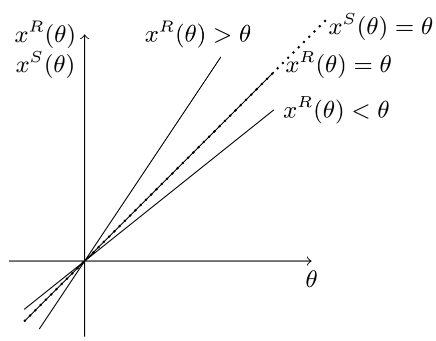

Figure 1: Conflicts with respect to θ between sender and receiver.

The difference $\theta - \mathbb{E}[\omega|\theta]$ describes the bias of the sender relative to the receiver. If the sender observes information that is relatively more informative about η, then the sender has incentives to exaggerate. If the sender's information is relatively more informative about ω, then the sender has incentives to downplay. Finally, there is no bias when communicating about the sender's inference when the sender's inference θ is equally informative about ω and η. For convenience, the three cases are depicted in Figure 1.

4 Optimal information structures

We now address headquarters' problem of choosing an optimal information structure. What information should the sender get to observe about each of the underlying taste parameters, ω and η? The sender's information impacts on payoffs through two channels. Firstly, assuming honest transmission of the sender's inference, the relative informational content of θ impacts directly on the sender's and the receiver's expected payoff from making a receiver-optimal decision based on θ. Secondly, the relative informational content determines the sender's bias in the communication game and thus impacts on the amount of information that is transmitted through communication. It is helpful to look at the two margins separately. Therefore, we begin our analysis with the clearly unrealistic case where the sender's inference θ becomes publicly available.[13] In a second step, in section 4.2, we look at the case of main interest, where θ is private information.

To streamline the exposition, we present our analysis first assuming that marginals are identical. That is, we assume $\sigma_\omega^2 = \sigma_\eta^2$. We discuss the role of this assumption and abandon it in section 4.4 below.

4.1 Public inferences

4.1.1 Headquarters problem

If the receiver observes the sender's inference θ, then he follows the policy $x^R(\theta) = \mathbb{E}[\omega|\theta] = \frac{Cov(\omega,\theta)}{Var(\theta)} \cdot \theta$, resulting in a loss of $\ell\left(\frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \omega\right)$ for the receiver and a loss of $\ell\left(\frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \eta\right)$

[13]We can think of this as some form of mediated information transmission; the sender's information s_ω, s_η is aggregated to $\alpha^S s_\omega + \beta^S s_\eta = \theta$ and then mechanically transmitted to the receiver.

for the sender. Both losses depend only on sums of the underlying random variables, $\zeta \equiv \frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \omega$ and $\tau \equiv \frac{Cov(\omega,\theta)}{Var(\theta)}\theta - \eta$, which are again elliptical (Normal). Let σ_ζ^2 and σ_τ^2 denote the variances of ζ and τ and let $z \equiv \frac{\zeta}{\sigma_\zeta}$ and $t \equiv \frac{\tau}{\sigma_\tau}$ denote the standardized arguments of the loss functions. As demonstrated formally in the appendix, we can write headquarters' problem as

$$\max_{Cov(\omega,\theta),Var(\theta)} - \int \ell(\sigma_\zeta z)\, c\phi(z)\, dz - \int \ell(\sigma_\tau t)\, c\phi(t)\, dt$$
$$s.t.\ Cov(\omega,\theta), Var(\theta) \text{ feasible.}$$

where z and t follow a spherical (Standard Normal) distribution with density $c\phi(\cdot)$.

Each division's expected utility depends negatively on a residual variance that measures the residual uncertainty after using θ optimally from the receiver's perspective. Naturally, the residual uncertainty for the receiver is

$$\sigma_\zeta^2 = \sigma_\omega^2 - \frac{Cov(\omega,\theta)^2}{Var(\theta)} = Var(\omega|\theta), \tag{6}$$

where the second equality holds because θ is used optimally from the receiver's perspective.[14] In contrast, θ is in general not used optimally from the sender's perspective. The residual uncertainty that the sender faces when θ is used according to the policy $x^R(\theta)$ is

$$\sigma_\tau^2 = \sigma_\eta^2 - \left(2Cov(\omega,\theta) - \frac{Cov(\omega,\theta)^2}{Var(\theta)}\right), \tag{7}$$

which differs from $Var(\eta|\theta) = \sigma_\eta^2 - Var(\theta)$ unless (5) and (4) are identically equal to each other.

Consider now the feasible set of information structures. Not any joint distribution of ω, η, θ is a feasible reduced form information structure, because θ must be derived from Bayesian updating by the sender about η, conditioning on the information that the sender gets to see. Thus, a joint distribution of ω, η and θ is feasible only if there are noise variances $\sigma_{\varepsilon_\omega}^2$ and $\sigma_{\varepsilon_\eta}^2$ that, together with the prior distribution, induce the joint distribution. The following lemma makes the restrictions from Bayesian updating explicit.

[14]For the derivation of the conditional second moments see Lemma A.1 in the appendix.

Lemma 3 *A joint distribution of ω, η, θ can be generated through Bayesian updating if and only if $Cov(\omega, \theta) \in [0, \sigma_{\omega\eta}]$ and for any given $Cov(\omega, \theta) = C$, $Var(\theta) \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho C, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C\right]$.*

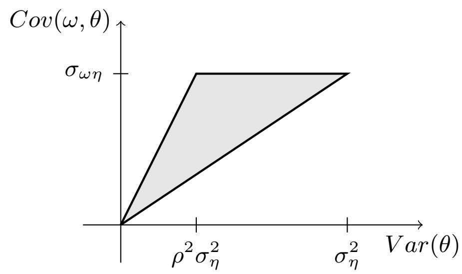

Figure 2: The feasible set of information structures, Γ.

$Var(\theta)$ and $Cov(\omega, \theta)$ are jointly constrained to lie in the triangle described in Figure 2. We call the feasible set Γ. To understand the shape of Γ, note that any pair of normalized noise variances, $\frac{\sigma^2_{\varepsilon\eta}}{\sigma^2_\eta}, \frac{\sigma^2_{\varepsilon\omega}}{\sigma^2_\omega} \geq 0$, results in a $Cov(\omega, \theta) \leq \sigma_{\omega\eta}$. The covariance is maximal if at least one of the signals is perfectly precise. In the limiting case of infinitely noisy signals, the sender does not revise his prior at all and so both $Var(\theta)$ and $Cov(\omega, \theta)$ are zero. If the sender observes a signal s_η without noise, $\sigma^2_{\varepsilon_\eta} = 0$, then his posterior mean becomes identically equal to η and the resulting variance is $Var(\theta) = \sigma^2_\eta$. If the sender observes s_ω without noise, $\sigma^2_{\varepsilon_\omega} = 0$, and the signal s_η is infinitely noisy, $\sigma^2_{\varepsilon_\eta} \to \infty$, then $Var(\theta) = \rho^2\sigma^2_\eta$, because the sender's posterior mean rises less than one for one with the sender's observation. By continuity, any pair of covariance and variance in the interior of the triangle can be generated by some pair of noise variances. Finally, Γ is always nonempty, because the lowest feasible $Var(\theta)$ for any given $Cov(\omega, \theta)$ is below the highest feasible $Var(\theta)$ by the Cauchy-Schwarz inequality, $\sigma^2_{\omega\eta} \leq \sigma^2_\eta\sigma^2_\omega$.[15]

[15] We include edges and vertices in the feasible set that result from taking limits. The limiting posterior distributions and moments when one noise variance goes out of bounds converge to the distribution when only one signal is received; the limiting case when both noise variances go out of bounds converges to the distribution when no signal at all is received, the prior.

4.1.2 Equalizing residual uncertainty

We can now restate headquarters' problem as

$$\max_{Cov(\omega,\theta),Var(\theta)} -\int \ell\left(\sigma_{\zeta} z\right) c\phi\left(z\right) dz - \int \ell\left(\sigma_{\tau} t\right) c\phi\left(t\right) dt$$
$$s.t.\ Cov\left(\omega,\theta\right), Var\left(\theta\right) \in \Gamma,$$

where σ_ζ and σ_τ are defined in (6) and (7). Headquarters maximizes a continuous objective function on a compact domain, so the problem is well defined and a solution exists. The solution takes the following form:

Theorem 1 *Suppose that the sender and the receiver are equally uncertain ex ante,* $\sigma_\omega^2 = \sigma_\eta^2$*. If the loss function satisfies* $\frac{q\ell''(q)}{\ell'(q)} > 1$ *for all* $q \neq 0$*, then headquarters' problem of choosing an optimal information structure has a unique solution, which is given by* $Var\left(\theta\right)^* = Cov\left(\omega,\theta\right)^* = \sigma_{\omega\eta}$*. If the loss function satisfies* $\frac{q\ell''(q)}{\ell'(q)} = 1$ *for all* $q \neq 0$ *(corresponding to the quadratic case), then any information structure satisfying* $Cov\left(\omega,\theta\right) = \sigma_{\omega\eta}$ *is optimal.*

We solve the problem by maximizing sequentially with respect to $Var\left(\theta\right)$ and $Cov\left(\omega,\theta\right)$. For a given level of $Cov\left(\omega,\theta\right)$, headquarters' problem resembles a risk sharing problem. Both divisions dislike higher residual uncertainty and an increase of $Var\left(\theta\right)$ increases (6), the residual uncertainty the receiver faces, and decreases (7), the residual uncertainty the sender faces. For a sufficiently convex loss function, the problem is single-peaked in $Var\left(\theta\right)$ and has a unique maximum at the point where the residual uncertainty for both divisions is equalized. Equating (6) and (7) and solving for $Var\left(\theta\right)$, we obtain

$$Var\left(\theta\right)^* = Cov\left(\omega,\theta\right).$$

The residual uncertainty for both divisions is then equal to the residual uncertainty that the receiver faces, $Var\left(\omega|\,\theta\right) = \sigma_\omega^2 - Cov\left(\omega,\theta\right)$. Since this is a decreasing function of $Cov\left(\omega,\theta\right)$, it is optimal to choose $Cov\left(\omega,\theta\right)$ as high as possible,

$$Cov\left(\omega,\theta\right)^* = \sigma_{\omega\eta}.$$

The unique optimum corresponds to the intersection of the dashed and the solid line in Figure 3. The role of the curvature condition is to guarantee uniqueness of the optimal

$Var\,(\theta)$. For the quadratic loss function, headquarters' payoff becomes linear in the residual variances, which implies that the receiver's loss from increasing $Var\,(\theta)$ just offsets the sender's gain and thus the sum of their payoffs becomes independent of $Var\,(\theta)$. Hence, any information structure with the highest feasible $Cov\,(\omega, \theta)$, depicted as the solid line in the figure, is optimal.

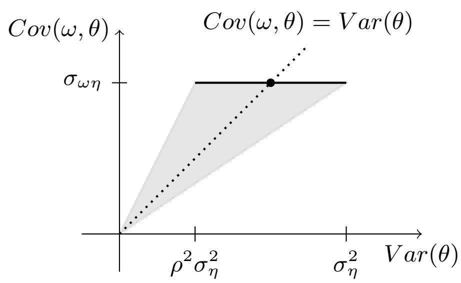

Figure 3: The optimal information structure maximizes $Cov(\omega, \theta)$. For sufficiently convex loss functions it is unique and satisfies $Cov(\omega, \theta) = Var(\theta)$.

The optimum can be understood by decomposing information into its common and idiosyncratic content. Since $Cov\,(\omega, \eta|\,\theta) = \sigma_{\omega\eta} - Cov\,(\omega, \theta)$, $Cov\,(\omega, \theta)$ measures the amount of common information. Naturally, the optimal information structure contains all the common information there is,

$$Cov\,(\omega, \eta|\,\theta)^* = \sigma_{\omega\eta} - Cov\,(\omega, \theta)^* = 0,$$

implying that conditional on θ, the taste parameters become uncorrelated. $Var\,(\theta)$ measures the amount of idiosyncratic information. Since there is only one signal, θ, idiosyncratic information necessarily involves a trade-off: $Var\,(\omega|\,\theta)$ is increasing in $Var\,(\theta)$, while $Var\,(\eta|\,\theta)$ is decreasing in $Var\,(\theta)$.

In terms of the underlying signals, headquarters allows the sender to observe ω without noise, $\sigma^2_{\varepsilon_\omega} = 0$, but adds noise $\sigma^2_{\varepsilon_\eta} = \frac{1-\rho^2}{\rho}\sigma^2_\eta$ to the signal about η. If the signal s_η were perfectly precise, then the sender would not pay any attention to the signal s_ω. While θ would still contain the maximum amount of common information, θ would not be informative enough about ω and so the receiver would face too much residual uncertainty. Hence, noise is needed to keep the sender from using the signal that is of primary importance to him exclusively.

4.2 Private inferences

We now consider the case of main interest where the sender has private information about θ and thus is free to make up any statement he likes. As is standard in the literature, we assume that the sender and the receiver are able to coordinate on the ex ante Pareto optimal equilibrium in the communication game. The optimal information structure eliminates conflicts in a certain, well defined sense:

Theorem 2 *Let the sender and the receiver face equal prior uncertainty, $\sigma_\omega^2 = \sigma_\eta^2$. Then, the unique optimal information structure chosen by headquarters satisfies $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$. The Pareto best equilibrium of the ensuing continuation game involves smooth strategies; the sender truthfully announces θ, $m^*(\theta) = \theta\ \forall\theta$, and the receiver takes the sender's advice at face value, $x^*(m) = m\ \forall m$. All parties' payoffs are the same as if the sender were given the right to choose the action x directly.*

The theorem is a straightforward implication of our preceding results in conjunction with a verification that the described strategies constitute an equilibrium of the communication game. Since headquarters cannot improve upon its payoff compared to the case where θ is public information, the situation corresponds to an optimum if this payoff is reached. Suppose the receiver believes that the sender plays the message strategy $m(\theta) = \theta$ for all θ. Then, his best reply is the action strategy $x^*(m) = \frac{Cov(\omega,\theta)^*}{Var(\theta)^*} \cdot m = m$ for all m. The sender, who anticipates this policy, induces his ideal policy by being truthful about θ, so the construction is indeed an equilibrium. Note that in this equilibrium the strategies of both players are smooth - in fact, linear - functions.

Since $x^*(m^*(\theta)) = \theta$ for all θ, the sender's optimal policy is implemented for all θ. Consequently, whether the sender communicates with the receiver or whether the sender is given the right to choose the policy, the payoffs of all parties involved are exactly the same.[16] The intuition is that, for equal marginals, an information structure that equalizes residual uncertainty automatically eliminates any bias in the use of information. Formally,

[16]Note that the problem of multiple solutions for the quadratic loss case if θ is public information is eliminated, because truthful communication now requires that $\frac{Cov(\omega,\theta)^*}{Var(\theta)^*} = 1$.

$Cov\left(\omega,\theta\right)^{*}=Var\left(\theta\right)^{*}$ implies

$$x^{R}\left(\theta\right)-x^{S}\left(\theta\right)=\left(\frac{Cov\left(\omega,\theta\right)^{*}}{Var\left(\theta\right)^{*}}-1\right)\cdot\theta=0\quad\forall\theta.$$

Note that there remains a conflict between sender and receiver with respect to using the underlying signals, s_{ω} and s_{η}. However, the receiver simply cannot do better than follow the sender's advice, because based on observing the sender's inference θ, a garbled piece of information, the receiver's ideal choice coincides with the sender's ideal choice based on observing the underlying signals. The sender is willing to share his inference despite disagreement too. The sender knows that the receiver would ideally like to choose an action that matches the state ω, not θ. However, under the optimal information structure, the sender's recommendation θ and the difference $\omega-\theta$ become uncorrelated. Put differently, the optimal information structure *orthogonalizes* the conflict between the divisions and the recommendation and hence removes any impediments to communication.

Communication is in fact unsurpassed by any form of delegation, even *optimal delegation*. Even if headquarters or the receiver had the right to constrain the sender's discretion under delegation, they would not want to make use of this right. The sender's optimal choice is necessarily a function of his inference θ only, and the sender uses this inference in the receiver's best interest. Hence, constraining the sender's discretion under delegation decreases the receiver's payoff and joint surplus.

4.3 The quality of decision making

Under the optimal information structure information is lost because only inferences are transmitted. How much is lost by such garbling and how does this depend on the underlying conflicts?

We can measure the amount of information transmitted in equilibrium by the variance of induced choices; the higher this variance, the more information is transmitted. Headquarters throws in just enough noise to ensure that $Var\left(\theta\right)=\sigma_{\omega\eta}$. For identical priors, the variance of the induced choice is thus

$$Var\left(\theta\right)=\rho\sigma_{\eta}^{2}.$$

The higher is ρ, the more variable the induced choice. In the limit as $\rho\to1$, the sender truthfully announces η and the variance of choices approaches σ_{η}^{2}. There are two reasons why increasing ρ results in an improvement of information transmission. Recall that the sender always observes ω without noise. The higher is ρ, the higher the attention the sender pays to this signal and the more this signal is reflected in the sender's preferred choice. Moreover, the sender observes η with an amount of noise equal to $\sigma^2_{\varepsilon_\eta} = \frac{1-\rho^2}{\rho}\sigma^2_\eta$, a decreasing function of ρ. The higher is ρ, the more precise the sender's signal about the sender-relevant random variable η. So, senders with better aligned interests are more trustworthy to begin with and get endowed with more precise information, rendering their advice even more valuable.[17]

4.4 Extensions: unequal priors

We now drop the assumption of equal prior uncertainty and allow for $\sigma^2_\omega \neq \sigma^2_\eta$. For quadratic loss functions, the canonical case studied in the literature, our result generalizes to asymmetric priors.

Proposition 1 *Assume quadratic loss functions and suppose that* $\min\left\{\sigma^2_\omega, \sigma^2_\eta\right\} \geq \sigma_{\omega\eta}$. *Then, headquarters' optimal choice of information structure is unique and given by* $Var(\theta)^* = Cov(\omega,\theta)^* = \sigma_{\omega\eta}$. *All parties receive the same expected payoff, regardless of who has the right to choose* x.

Recall that by Theorem 1 any information structure satisfying $Cov(\omega,\theta) = \sigma_{\omega\eta}$ is optimal for quadratic losses if θ is public. Hence, to show that headquarters can reach the same expected payoff under communication of unverifiable information - and under delegation - it suffices to show that the admissible set of information structures contains the element $Var(\theta) = Cov(\omega,\theta) = \sigma_{\omega\eta}$. The condition in the proposition is equivalent to

$$\frac{\sigma_\eta}{\sigma_\omega}\rho \leq 1 \leq \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho},$$

which guarantees that the 45° line is an element of the feasible set, Γ. We need to rule out very asymmetric priors where $\sigma^2_\omega > \sigma_{\omega\eta} > \sigma^2_\eta$ or $\sigma^2_\eta > \sigma_{\omega\eta} > \sigma^2_\omega$ that would render the

[17] In the limit, the feasible set of information structures converges to the 45° line and any piece of information is equally informative about ω and η. Hence endowing the sender with perfect information becomes optimal. At the same time the underlying interests of sender and receiver become perfectly correlated.

solution $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$ infeasible. By Cauchy-Schwarz, it is impossible that both prior variances exceed the covariance, so the restriction is quite mild.

Note that nonverifiability makes the solution unique. While the sum of residual variances is constant for all information structures with the highest feasible $Cov(\omega, \theta)$, there is only one information structure among them that makes the signal θ equally useful for both sender and receiver and thus ensures that truthful communication about θ is an equilibrium.

5 Conclusions

Two divisions, overarched by a headquarters, need to reach a decision that affects the payoffs of all parties involved. Division one privately gets to observe information about ideal policies from both divisions' perspectives. Division one draws inferences from the information and communicates them to division two. Division two, who retains the right to make the decision, draws its own inferences from division one's inferences. Anticipating the chain of inferences within the organization, headquarters chooses what information to acquire at the outset. Choosing what to look into is a powerful tool. When properly done, conflicts within the organization are diminished, making it less important who has the right to make decisions: communication and delegation become outcome equivalent.

Almost by definition, an equivalence result raises nearly as many questions as it answers. In particular, one may wonder what happens if not headquarters but the sender has discretion over the acquisition of information. Quite clearly, it seems, that the benchmark of no loss through communication cannot be reached. Much to our surprise, we show in companion work that this conclusion is unwarranted. For a special case of the current environment, we are able to show that the sender acquiring orthogonalized information remains an equilibrium of the game. There are also other equilibria, but the sender cannot gain from making the information more useful to himself - precisely, because its usefulness would be lost in communication. Many other interesting questions can be pursued in our environment. We leave these for future work.

A Appendix

Lemma A.1 *Let Y follow an elliptical distribution, $Y \sim EC_n(\mu, \Sigma, \phi)$. Further let*

$$Y = (Y_1, Y_2), \quad \mu = (\mu_1, \mu_2), \quad \Sigma = \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix},$$

where the dimensions of Y_1, μ_1 and Σ_{11} are m, m, and $m \times m$.

i) The elliptical distribution is symmetric about μ.

ii) Linear combinations of elliptically distributed random variables are again elliptical.

iii) The conditional distribution of $(Y_1|Y_2 = y_2)$ is elliptical, with conditional mean vector

$$\mathbb{E}[Y_1|Y_2 = y_2] = \mu_1 + (y_2 - \mu_2) \Sigma_{22}^{-1} \Sigma_{21} \tag{A1}$$

and conditional covariance matrix satisfying

$$\Sigma^* = \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{21}. \tag{A2}$$

Proof of Lemma A.1. i) by definition, ii) Fang et al. (1990) Theorem 2.16, iii) Fang et al. (1990) Theorem 2.18. ■

Proof of Lemma 1. Let $u \equiv u^R = u^S$ and $z = \omega, \eta$. Consider the problem

$$\max_x \int_{-\infty}^{\infty} u(x - z) f(z | s_\omega, s_\eta) \, dz,$$

where $f(z | s_\omega, s_\eta)$ is the conditional density of $z = \omega, \eta$ given the signals. Since the utility depends only on the distance between x and z we have $u'(x - z) > 0$ for $z < x$, $u'(x - z) = 0$ for $x = z$, and $u'(x - z) < 0$ for $z > x$.

Consider the candidate solution $x^* = \mu_z \equiv \mathbb{E}[z | s_\omega, s_\eta]$. The first-order condition can be written as

$$\int_{-\infty}^{\infty} u'(x^* - z) f(z | s_\omega, s_\eta) \, dz = \int_{-\infty}^{\infty} u'(\mu_z - z) f(z | s_\omega, s_\eta) \, dz = 0.$$

Consider two points $z_1 = \mu_z - \Delta$ and $z_2 = \mu_z + \Delta$ for arbitrary $\Delta > 0$. By symmetry of u around its bliss point and symmetry of the distribution around μ_z, we have

$$u'(\Delta) f(\mu_z - \Delta | s_\omega, s_\eta) = -u'(-\Delta) f(\mu_z + \Delta | s_\omega, s_\eta).$$

Since this holds point-wise for each Δ, it also holds if we integrate over Δ. Thus, the first-order condition is satisfied at $x^* = \mu_z$. By concavity of u in x, only one value of x satisfies the first-order condition.

Applying equation $(A1)$, the conditional expectations are

$$\mathbb{E}[\eta|s_\omega, s_\eta] = \alpha^S s_\omega + \beta^S s_\eta \tag{A3}$$

and

$$\mathbb{E}[\omega|\, s_\omega, s_\eta] = \alpha^R s_\omega + \beta^R s_\eta, \tag{A4}$$

where the weights in the sender's ideal choice are

$$\alpha^S = \sigma^2_{\varepsilon_\eta} \frac{\rho\sigma_\omega\sigma_\eta}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma_\omega\sigma_\eta)^2}$$

and

$$\beta^S = \sigma^2_\eta \frac{\sigma^2_{\varepsilon_\omega} - \sigma^2_\omega\rho^2 + \sigma^2_\omega}{(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega})(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}) - (\rho\sigma_\omega\sigma_\eta)^2}$$

and the weights in the receiver's ideal choice are

$$\alpha^R = \sigma^2_\omega \frac{\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta\rho^2}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - \left(\rho\sigma_\omega\sigma_\eta\right)^2}$$

and

$$\beta^R = \sigma^2_{\varepsilon_\omega} \frac{\sigma_\eta\sigma_\omega\rho}{\left(\sigma^2_\omega + \sigma^2_{\varepsilon_\omega}\right)\left(\sigma^2_\eta + \sigma^2_{\varepsilon_\eta}\right) - \left(\rho\sigma_\omega\sigma_\eta\right)^2}.$$

First, suppose $\sigma^2_{\varepsilon_\eta}$ and $\sigma^2_{\varepsilon_\omega}$ are both positive and finite. Equations $(A3)$ and $(A4)$ are identical for all s_ω and s_η if and only if

$$\sigma^2_{\varepsilon_\eta}\rho\sigma_\omega\sigma_\eta = \sigma^2_\omega\left(\sigma^2_{\varepsilon_\eta} + \sigma^2_\eta - \sigma^2_\eta\rho^2\right)$$

and

$$\sigma^2_\eta\left(\sigma^2_{\varepsilon_\omega} - \sigma^2_\omega\rho^2 + \sigma^2_\omega\right) = \sigma_\eta\sigma_\omega\rho\sigma^2_{\varepsilon_\omega}.$$

This requires that

$$\sigma^2_\eta\left(1 - \rho^2\right) = \left(\frac{\rho\sigma_\eta}{\sigma_\omega} - 1\right)\sigma^2_{\varepsilon_\eta}$$

and

$$\sigma_\omega^2\left(1-\rho^2\right)=\left(\frac{\sigma_\omega\rho}{\sigma_\eta}-1\right)\sigma_{\varepsilon_\omega}^2.$$

A necessary and sufficient condition for these two conditions to hold simultaneously is $\sigma_{\omega\eta}=\sigma_\eta^2=\sigma_\omega^2$.

Consider now the limiting cases where one of the variances goes out of bounds. Applying l'Hôpital's rule to $(A3)$ and $(A4)$, we get in the limit as $\sigma_{\varepsilon_\eta}^2\to\infty$

$$\mathbb{E}\left[\omega|s_\omega\right]=\frac{\sigma_\omega^2}{\sigma_\omega^2+\sigma_{\varepsilon_\omega}^2}s_\omega \qquad \text{and} \qquad \mathbb{E}\left[\eta|s_\omega\right]=\frac{\rho_{\omega\eta}\sigma_\omega\sigma_\eta}{\sigma_\omega^2+\sigma_{\varepsilon_\omega}^2}s_\omega,$$

so that

$$\mathbb{E}\left[\omega|s_\omega\right]\equiv\mathbb{E}\left[\eta|s_\omega\right] \qquad \Leftrightarrow \qquad \rho\sigma_\eta=\sigma_\omega.$$

Likewise, for the case where $\sigma_{\varepsilon_\omega}^2\to\infty$, we get

$$\mathbb{E}\left[\omega|s_\eta\right]=\frac{\rho\sigma_\omega\sigma_\eta}{\sigma_\eta^2+\sigma_{\varepsilon_\eta}^2}s_\eta \qquad \text{and} \qquad \mathbb{E}\left[\eta|s_\eta\right]=\frac{\sigma_\eta^2}{\sigma_\eta^2+\sigma_{\varepsilon_\eta}^2}s_\eta,$$

so

$$\mathbb{E}\left[\omega|s_\eta\right]\equiv\mathbb{E}\left[\eta|s_\eta\right] \qquad \Leftrightarrow \qquad \rho\sigma_\omega=\sigma_\eta.$$

■

Proof of Lemma 2. Let $u\equiv u^R=u^S$. Recall from Lemmas 1 and A.1 that $\theta=\mathbb{E}\left[\eta|s_\omega,s_\eta\right]$ and that the conditional distribution of η given s_ω,s_η is symmetric about θ. We first show that the sender's preferences over messages depend only on the distance between induced actions and θ. Let $x'-\mathbb{E}\left[\eta|\,s_\omega,s_\eta\right]=\mathbb{E}\left[\eta|\,s_\omega,s_\eta\right]-x''\equiv z>0$, then

$$\int u\left(x'-\eta\right)f\left(\eta|\,s_\omega,s_\eta\right)d\eta=\int u\left(z-\left(\eta-\mathbb{E}\left[\eta|\,s_\omega,s_\eta\right]\right)\right)f\left(\eta|\,s_\omega,s_\eta\right)d\eta.$$

The random variable $\hat{\eta}\equiv\eta-\mathbb{E}\left[\eta|\,s_\omega,s_\eta\right]$ has mean zero and follows a symmetric distribution. Let $\hat{f}\left(\hat{\eta}|\,s_\omega,s_\eta\right)$ denote the standardized distribution (with mean zero). Then, we have

$$f\left(\eta|\,s_\omega,s_\eta\right)=\hat{f}\left(\eta-\mathbb{E}\left[\eta|\,s_\omega,s_\eta\right]|\,s_\omega,s_\eta\right)=\hat{f}\left(\hat{\eta}|\,s_\omega,s_\eta\right).$$

Take two realizations $\hat{\eta}'$ and $\hat{\eta}''=-\hat{\eta}'$ of $\hat{\eta}$. By construction, we have $|z-\hat{\eta}'|=|-z-\hat{\eta}''|$ and hence by symmetry of u around 0, $u\left(z-\hat{\eta}'\right)=u\left(-z-\hat{\eta}''\right)$. Symmetry of the distribution around zero is equivalent to $\hat{f}\left(\hat{\eta}'|\,s_\omega,s_\eta\right)=\hat{f}\left(\hat{\eta}''|\,s_\omega,s_\eta\right)$. Therefore, for all $\hat{\eta}'$ we have

$u(z-\hat{\eta}')\hat{f}(\hat{\eta}'|\, s_\omega, s_\eta) = u(\hat{\eta}'-z)\hat{f}(\hat{\eta}'|\, s_\omega, s_\eta)$, implying that

$$\int u(z-\hat{\eta})\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta} = \int u(\hat{\eta}-z)\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta}.$$

By symmetry of the distribution, $\hat{\eta}$ and $-\hat{\eta}$ follow the exact same distribution, and we can write

$$\int u(\hat{\eta}-z)\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta} = \int u(-z-\hat{\eta})\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta}.$$

Hence,

$$\begin{aligned}\int u(z-\hat{\eta})\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta} &= \int u(-z-\hat{\eta})\hat{f}(\hat{\eta}|\, s_\omega, s_\eta)\, d\hat{\eta} \\ &= \int u(-z-(\eta-\mathbb{E}[\eta|\, s_\omega, s_\eta]))\, f(\eta|\, s_\omega, s_\eta)\, d\eta \\ &= \int u(x''-\eta)\, f(\eta|\, s_\omega, s_\eta)\, d\eta,\end{aligned}$$

that is, the sender is indifferent between actions that are equidistant from θ. By concavity of the sender's payoff, the sender prefers action x' over x'' if and only if x' is closer to θ.

Suppose now that an equilibrium transmits more information to the receiver than θ. Then it must be that the sender is indifferent between the induced actions, x', x'', that is they must satisfy $|\theta - x'| = |x'' - \theta|$.

Suppose for sender type θ, the equilibrium induces two actions, equidistant from θ, with some distance $\varepsilon > 0$. Now take a type $\tilde{\theta} = \theta + \delta$ for some $\delta > 0$. We distinguish three cases. Suppose first type $\tilde{\theta}$ induces one action $\overline{x}(\tilde{\theta}) \geq \tilde{\theta}$. Then, to discourage any deviation, we need to have

$$\underline{x}(\theta) < \theta < \overline{x}(\theta) \leq \tilde{\theta} \leq \overline{x}(\tilde{\theta}). \tag{A5}$$

However, for all $\delta < \varepsilon$ we have $\overline{x}(\theta) = \theta + \varepsilon > \theta + \delta = \tilde{\theta}$ contradicting condition ($A5$) and implying that some types have a strict incentive to lie. If type $\tilde{\theta}$ induces one action $\underline{x}(\tilde{\theta}) \leq \tilde{\theta}$, or two actions that are equidistant from $\tilde{\theta}$ and satisfy $\underline{x}(\tilde{\theta}) \leq \tilde{\theta} \leq \overline{x}(\tilde{\theta})$, then condition (A5) needs to be amended to

$$\underline{x}(\theta) < \theta < \overline{x}(\theta) \leq \underline{x}(\tilde{\theta}) \leq \tilde{\theta}\left(\leq \overline{x}(\tilde{\theta})\right),$$

where the last inequality is absent if type $\tilde{\theta}$ induces only one action $\underline{x}(\tilde{\theta}) \leq \tilde{\theta}$. Since the new condition is even more difficult to satisfy than (A5), the same reasoning applies. ■

Proof of Lemma 3. Letting $a \equiv \frac{\sigma_{\varepsilon\omega}^2}{\sigma_\omega^2}$ and $b \equiv \frac{\sigma_{\varepsilon\eta}^2}{\sigma_\eta^2}$ we can rewrite $Cov(\omega, \theta)$ and $Var(\theta)$ as

$$Cov(\omega, \theta) = \sigma_{\omega\eta} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2},$$

and

$$Var(\theta) = \sigma_\eta^2 \frac{a + b\rho^2 + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2}.$$

Consider first the set of feasible levels of $Cov(\omega, \theta) = C$. Note that for $a = 0$ or $b = 0$, the covariance is constant and equal to $\sigma_{\omega\eta}$. Moreover, the covariance is decreasing in a for given b and decreasing in b for given a. By l'Hôpital's rule, we have

$$\lim_{b\to\infty} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2} = \frac{1}{1 + a},$$

and

$$\lim_{a\to\infty} \frac{a + b + 1 - \rho^2}{(1 + a)(1 + b) - \rho^2} = \frac{1}{1 + b}.$$

So, letting both a and b (in whatever order) go to infinity results in a covariance of zero. By continuity, any $C \in (0, \sigma_{\omega\eta}]$ can be generated by finite levels a, b. Including the case where no signal is observed at all, we can generate all $C \in [0, \sigma_{\omega\eta}]$.

Consider next the set of feasible $Var(\theta)$ for any given level $Cov(\omega, \theta) = C$. Distinguish two cases, i) $C = \sigma_{\omega\eta}$ and ii) $C \in [0, \sigma_{\omega\eta})$.

Case i) requires that $a = 0$ or $b = 0$ or both. If $b = 0$, then $\frac{a+b\rho^2+1-\rho^2}{(1+a)(1+b)-\rho^2} = 1$ and thus $Var(\theta) = \sigma_\eta^2$ for all a. If $a = 0$, then

$$Var(\theta) = \sigma_\eta^2 \frac{b\rho^2 + 1 - \rho^2}{(1 + b) - \rho^2}$$

is decreasing in b and attains value $Var(\theta) = \sigma_\eta^2$ for $b = 0$. Moreover,

$$\lim_{b\to\infty} \frac{b\rho^2 + 1 - \rho^2}{(1 + b) - \rho^2} = \rho^2.$$

Hence, for $C = \sigma_{\omega\eta}$, $Var(\theta) \in \left[\rho^2\sigma_\eta^2, \sigma_\eta^2\right]$; the lower limit is included because we allow for the case where only one signal is observed.

Case ii) $C \in [0, \sigma_{\omega\eta})$ requires that $a > 0$ and $b > 0$. Let $\gamma \equiv \frac{C}{\sigma_{\omega\eta}} \in [0,1)$. The combinations of a and b that generate C satisfy

$$\frac{a+b+1-\rho^2}{(1+a)(1+b)-\rho^2} = \gamma.$$

Solving for a as a function of b, we obtain

$$a(b;\gamma) = \frac{(1-\gamma)(1+b-\rho^2)}{\gamma b-(1-\gamma)} = \frac{(1+b-\rho^2)}{\frac{\gamma}{1-\gamma}b-1}.$$

The function $a(b;\gamma)$ is decreasing in b and has the limit

$$\lim_{b\to\infty} \frac{1+b-\rho^2}{\frac{\gamma}{1-\gamma}b-1} = \frac{1-\gamma}{\gamma}.$$

In the limit as $b \to \frac{1-\gamma}{\gamma}$, we obtain $a \to \infty$. Hence, C can be generated for $b > \frac{1-\gamma}{\gamma}$ and $a = \frac{(1+b-\rho^2)}{\frac{\gamma}{1-\gamma}b-1}$. Substituting for $\frac{(1+b-\rho^2)}{\frac{\gamma}{1-\gamma}b-1}$ into $Var(\theta)$, we obtain

$$Var(\theta; b, a(b;\gamma), \gamma) = \sigma_\eta^2 \frac{\frac{(1+b-\rho^2)}{\frac{\gamma}{1-\gamma}b-1} + b\rho^2 + 1 - \rho^2}{\left(1+\frac{(1+b-\rho^2)}{\frac{\gamma}{1-\gamma}b-1}\right)(1+b)-\rho^2} = \sigma_\eta^2 \frac{b\gamma\rho^2+1-\rho^2}{1+b-\rho^2}.$$

The derivative of this expression in b is $\frac{(\gamma\rho^2-1)(1-\rho^2)}{(1+b-\rho^2)^2} < 0$, so $Var(\theta; b, a(b;\gamma), \gamma)$ is continuous and monotone decreasing in b. In the limit as b tends to infinity, we obtain

$$\lim_{b\to\infty} \sigma_\eta^2 \frac{b\gamma\rho^2+1-\rho^2}{1+b-\rho^2} = \sigma_\eta^2\gamma\rho^2 = \sigma_\eta^2 \frac{C}{\sigma_{\omega\eta}}\rho^2 = \frac{\sigma_\eta}{\sigma_\omega}\rho C.$$

In the limit as $b \to \frac{1-\gamma}{\gamma}$, we obtain

$$\lim_{b\to\frac{1-\gamma}{\gamma}} \sigma_\eta^2 \frac{b\gamma\rho^2+1-\rho^2}{1+b-\rho^2} = \sigma_\eta^2 \frac{\frac{1-\gamma}{\gamma}\gamma\rho^2+1-\rho^2}{1+\frac{1-\gamma}{\gamma}-\rho^2} = \gamma\sigma_\eta^2 = \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C.$$

Hence, we have shown that for any given $C \in [0, \sigma_{\omega\eta})$, $Var(\theta) \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho C, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}C\right]$. We include the lower limit, because the case where $b \to \infty$ is equivalent to the case with one signal only.

■

Proof of Theorem 1. Let $u \equiv u^R = u^S$, $C \equiv Cov(\omega, \theta)$, and $V \equiv Var(\theta)$. We prove the theorem in two steps. In step i) we derive the standardized distributions. In step ii) we solve the maximization problem.

i) Let $f_{\omega\theta}(\omega, \theta) = \int f(\omega, \eta, \theta)\, d\eta$ and let $f_{\eta\theta}(\omega, \theta) = \int f(\omega, \eta, \theta)\, d\omega$ denote the marginal joint densities of ω, θ and η, θ. Consider first the expected utility of the sender.

Let $\tau \equiv \frac{C}{V}\theta - \eta$ and let $g(\cdot)$ denote the density of τ. The expected utility of the sender satisfies

$$\begin{aligned} & \int\int u\left(\frac{C}{V}\theta - \eta\right) f_{\eta\theta}(\eta, \theta)\, d\eta d\theta = \int\int u(\tau) f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\tau d\theta \\ = & \int u(\tau) \int f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\theta d\tau = \int u(\tau)\, g(\tau)\, d\tau = \int u(\sigma_\tau t)\, c\phi(t)\, dt. \end{aligned}$$

For the first equality, substitute τ and apply the switch of variables theorem. For the second, apply Fubini's theorem. For the third, note that $\Pr\left[\frac{C}{V}\theta - \eta \le \tau\right] = \Pr\left[\frac{C}{V}\theta - \tau \le \eta\right]$ and that by Leibniz's rule

$$g(\tau) = \frac{\partial}{\partial\tau} \Pr\left[\frac{C}{V}\theta - \eta \le \tau\right] = \frac{\partial}{\partial\tau} \int_{-\infty}^{\infty} \int_{\frac{C}{V}\theta - \tau}^{\infty} f_{\eta\theta}(\eta, \theta)\, d\eta d\theta = \int_{-\infty}^{\infty} f_{\eta\theta}\left(\frac{C}{V}\theta - \tau, \theta\right) d\theta.$$

Since τ is a linear function of θ and η, we can use Fang et al. (1990) Theorem 2.16 to conclude that $g(\tau)$ is the density of an elliptical distribution that has the same characteristic generator, $\phi(\cdot)$, as f has. The variance of τ is

$$\begin{aligned} \sigma_\tau^2 &= \frac{C^2}{V^2} Var(\theta) - 2\frac{C}{V} Cov(\theta, \eta) + Var(\eta) \\ &= \frac{C^2}{V} - 2C + \sigma_\eta^2. \end{aligned}$$

Standardizing to $t = \frac{\tau}{\sigma_\tau}$, we transform to a spherical (standardized elliptical) distribution with density $c\phi(\cdot)$.

To derive the receiver's expected utility, we let $\zeta \equiv \frac{C}{V}\theta - \omega$, let $h(\cdot)$ denote the density of ζ. Going through the exact same steps one finds that $h(\zeta) = \int f\left(\frac{C}{V}\theta - \zeta, \theta\right) d\theta$, again an elliptical density with the same characteristic generator. The variance of ζ is

$$\sigma_\zeta^2 = \sigma_\omega^2 - \frac{C^2}{V}.$$

Hence, with $z = \frac{\zeta}{\sigma_\zeta}$, we can write

$$\int \int u \left(\frac{C}{V} \theta - \omega \right) f_{\omega\theta} (\omega, \theta) \, d\omega d\theta = \int u \left(z \sigma_{\omega|\theta} \right) c\phi (z) \, dz.$$

ii) An optimal information structure solves:

$$\max_{C,V} \int u \left(z \left(\sigma_\omega^2 - \frac{C^2}{V} \right)^{\frac{1}{2}} \right) c\phi (z) \, dz + \int u \left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2 \right)^{\frac{1}{2}} t \right) c\phi (t) \, dt.$$

We solve the problem by maximizing sequentially wrt C and V. For given C, the derivative wrt V is

$$\begin{aligned} &\frac{1}{2} \frac{C^2}{V^2} \int z \left(\sigma_\omega^2 - \frac{C^2}{V} \right)^{-\frac{1}{2}} u' \left(z \left(\sigma_\omega^2 - \frac{C^2}{V} \right)^{\frac{1}{2}} \right) c\phi (z) \, dz \\ &- \frac{1}{2} \frac{C^2}{V^2} \int \left(\frac{C^2}{V} - 2C + \sigma_\eta^2 \right)^{-\frac{1}{2}} t u' \left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2 \right)^{\frac{1}{2}} t \right) c\phi (t) \, dt. \end{aligned} \tag{A6}$$

Recall that $\sigma_\eta^2 = \sigma_\omega^2$. First, suppose $V = C$. Then, the derivative wrt V satisfies

$$\begin{aligned} & \int \frac{1}{2} z \left(\sigma_\omega^2 - V \right)^{-\frac{1}{2}} u' \left(z \left(\sigma_\omega^2 - V \right)^{\frac{1}{2}} \right) c\phi (z) \, dz \\ & - \int \frac{1}{2} \left(-V + \sigma_\eta^2 \right)^{-\frac{1}{2}} t u' \left(\left(-V + \sigma_\eta^2 \right)^{\frac{1}{2}} t \right) c\phi (t) \, dt \\ = \; & 0. \end{aligned}$$

Now suppose $V \neq C$. Note that both integrands in (A6) have the common representation

$$\int \frac{1}{a} k u' (ak) \, c\phi (k) \, dk. \tag{A7}$$

Differentiating wrt a, we observe that (A7) is monotone decreasing in a,

$$-\frac{1}{a^3} \int aku' (ak) \, c\phi (k) \, dk + \frac{1}{a^3} \int a^2 k^2 u'' (ak) \, c\phi (k) \, dk \leq 0,$$

where the inequality follows from the curvature condition

$$q \frac{u'' (q)}{u' (q)} = q \frac{\ell'' (q)}{\ell' (q)} \geq 1. \tag{A8}$$

$V < C$ implies $\frac{C^2}{V} - 2C + \sigma_\eta^2 > \sigma_\omega^2 - \frac{C^2}{V}$. The curvature condition (A8) implies monotonicity and therefore

$$\begin{aligned} & \frac{1}{2}\frac{C^2}{V^2}\int z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - \frac{C^2}{V}\right)^{\frac{1}{2}}\right) c\phi(z)\, dz \\ \geq\ & \frac{1}{2}\frac{C^2}{V^2}\int \left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{-\frac{1}{2}} tu'\left(\left(\frac{C^2}{V} - 2C + \sigma_\eta^2\right)^{\frac{1}{2}} t\right) c\phi(t)\, dt. \end{aligned}$$

Hence the derivative is non-negative for $V < C$. By symmetry, the derivative is non-positive for $V > C$. These inequalities become strict for functions that satisfy the curvature condition (A8) with strict inequality. It follows that the problem is maximized in V for $V = C$.

The second step is now to maximize over C, given that $V = C$.

$$\max_C \int u\left(z\left(\sigma_\omega^2 - C\right)^{\frac{1}{2}}\right) c\phi(z)\, dz + \int u\left(\left(\sigma_\eta^2 - C\right)^{\frac{1}{2}} t\right) c\phi(t)\, dt.$$

The derivative wrt C is given by

$$\begin{aligned} & -\int \frac{1}{2} z\left(\sigma_\omega^2 - V\right)^{-\frac{1}{2}} u'\left(z\left(\sigma_\omega^2 - V\right)^{\frac{1}{2}}\right) c\phi(z)\, dz \\ & -\int \frac{1}{2}\left(\sigma_\eta^2 - C\right)^{-\frac{1}{2}} tu'\left(\left(\sigma_\eta^2 - C\right)^{\frac{1}{2}} t\right) c\phi(t)\, dt \\ >\ & 0. \end{aligned}$$

The payoff is unambiguously increasing in C. The solution is thus $C = C^{\max}$. ■

Proof of Proposition 1. By Theorem 1, for quadratic loss functions all information structures satisfying $Cov(\omega, \theta) = \sigma_{\omega\eta}$ are optimal for θ public. By Theorem 2, smooth communication is an equilibrium if and only if $Cov(\omega, \theta) = Var(\theta)$. By Lemma 3, the candidate solution $Var(\theta)^* = Cov(\omega, \theta)^* = \sigma_{\omega\eta}$ is feasible if $\frac{Cov(\omega,\theta)^*}{Var(\theta)^*} = 1 \in \left[\frac{\sigma_\eta}{\sigma_\omega}\rho, \frac{\sigma_\eta}{\sigma_\omega}\frac{1}{\rho}\right]$. This is guaranteed by the assumption $\min\left\{\sigma_\omega^2, \sigma_\eta^2\right\} \geq \sigma_{\omega\eta}$. ■

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